MCA TEST SERIES-6. Solution

Answer *with* Explanations

1. (*b*) Given, $A=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

As we know that, $A\cdot A^{-1}=I$

$$\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\Rightarrow$ $\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ is the inverse of A.

2. (*d*) Given system of equation is

$x+2y+3z=1,\ x-y+4z=0$ and $2x+y+7z=1.$

$$\Rightarrow \begin{vmatrix}1 & 2 & 3\\ 1 & -1 & 4\\ 2 & 1 & 7\end{vmatrix}=D,\quad D_1=\begin{vmatrix}1 & 2 & 3\\ 0 & -1 & 4\\ 1 & 1 & 7\end{vmatrix}$$

$$D_2=\begin{vmatrix}1 & 1 & 3\\ 1 & 0 & 4\\ 2 & 1 & 7\end{vmatrix} \text{ and } D_3=\begin{vmatrix}1 & 2 & 1\\ 1 & -1 & 0\\ 2 & 1 & 1\end{vmatrix}$$

Here, $D=D_1=D_2=D_3=0$

$\therefore$ The system has infinitely many solutions.

3. (*a*) $\begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y x & 1 & \log_y z\\ -\log_z x & \log_z y & 1\end{vmatrix}$

$$=\begin{vmatrix}\log_x x & \log_x y & \log_x z\\ \log_y x & \log_y y & \log_y z\\ -\log_z x & \log_z y & \log_z z\end{vmatrix}=\begin{vmatrix}\frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x}\\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y}\\ -\frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z}\end{vmatrix}$$

$$=\frac{1}{\log x\log y\log z}\begin{vmatrix}\log x & \log y & \log z\\ \log x & \log y & \log z\\ -\log x & \log y & \log z\end{vmatrix}$$

$$=\frac{1}{\log x\log y\log z}\times 0=0$$

4. (*b*) Equation of circle through (0, 0) is

$$x^2+y^2+2gx+2fy=0$$

Its radius is $\sqrt{g^2+f^2}$. $\quad [\because C=0]$

$\Rightarrow\quad \sqrt{g^2+f^2}=\sqrt{(1-g)^2+f^2}$

$(g^2+f^2)=1-2g+g^2+f^2\quad [\because$ (1, 0) lies on it$]$

$\Rightarrow\quad g=\frac{1}{2}$

Also, both circles touch each other.

$\therefore$ Distance between centres $=r_1+r_2$

$\Rightarrow\quad -\sqrt{g^2+f^2}=3+\sqrt{g^2+f^2}$

$\Rightarrow\quad -2\sqrt{g^2+f^2}=3$

$$g^2+f^2=\frac{9}{4}$$

$\Rightarrow\quad f^2=\frac{9}{4}-\frac{1}{4}\qquad [\because g=\frac{1}{2}]$

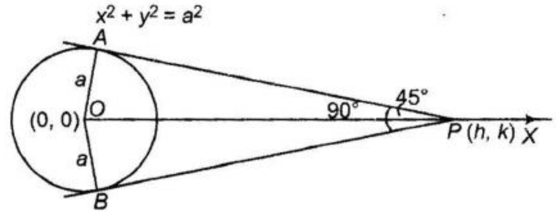
$\Rightarrow\quad f^2=\frac{8}{4}=2$

$\Rightarrow\quad f=\pm\sqrt{2}\Rightarrow f=-\sqrt{2}$

Hence, centre is $\left(\frac{1}{2},-\sqrt{2}\right)$.

5. (*b*) Let PA and PB be two perpendicular tangents from $P(h,k)$ to the circle $x^2+y^2=a^2$, then

$$\angle APO=45^\circ$$



$$\angle APO=45^\circ$$

$\Rightarrow\quad \tan 45^\circ=\frac{OA}{PA}$

$\Rightarrow\quad 1=\frac{a}{\sqrt{h^2+k^2-a^2}}$

$[\because OA=$ radius of the circle and $PA=$ length of tangent$]$

$\Rightarrow\quad h^2+k^2-a^2=a^2$

$\Rightarrow\quad h^2+k^2=2a^2$

$\therefore\quad$ The locus is $x^2+y^2=2a^2$.

6. (*b*) $(x^2+y^2-6x-6y+4)+\lambda(-4x-2y+1)=0$

$x^2+y^2-2x(3+2\lambda)-2y(3+\lambda)+(4+\lambda)=0$

Centre $=(3+2\lambda,3+\lambda)$

$$r=\sqrt{(3+2\lambda)^2+(3+\lambda)^2-(4+\lambda)}=0$$

$\Rightarrow 9+4\lambda^2+12\lambda+9+6\lambda+\lambda^2-4-\lambda=0$

$\Rightarrow\quad 5\lambda^2+17\lambda+14=0$

$\Rightarrow\quad 5\lambda^2+10\lambda+7\lambda+14=0$

$\Rightarrow\quad (5\lambda+7)(\lambda+2)=0$

$\Rightarrow\quad \lambda=-2,-7/5$

$\therefore$ Limiting points are $(-1,1)$ and $\left(3-\frac{14}{5},3-7/5\right)$ or $(-1,1)$ and $(1/5, 8/5)$.

7. (*c*) The two given circles are $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$.

Let the required circle be $x^2+y^2+2gx+2fy+c=0$.

$\because\quad 2g_1x+2f_1y+c_1=0$

$\quad\ \ 2g_2x+2f_2y+c_2=0$

$$2gg_1 + 2ff_1 = c_1 + c \quad \ldots(i)$$

and $$2gg_2 + 2ff_2 = c_2 + c \quad \ldots(ii)$$

[Since, the circle is orthogonal]

On subtracting Eq. (ii) from Eq. (i), we get

$$2g(g_1 - g_2) + 2f(f_1 - f_2) = c_1 - c_2$$

Hence, locus of required circle is radical axis of the given circles.

8. (c) Obviously, the rolling circle describes a circle concentric with the given circle and radius increases by 2.

So, equation of circle will be

$$x^2 + y^2 + 3x - 6y - 31 = 0$$

9. (d) On putting $x = \frac{y^2}{8}$ in $4x - 3y + 4 = 0$, we get

$$4 \times \frac{y^2}{8} - 3y + 4 = 0$$

$\Rightarrow$ $$y^2 - 6y + 8 = 0$$

$\Rightarrow$ $$y = 4, 2$$

$\Rightarrow$ $$x = \frac{16}{8} = 2$$

and $$x = \frac{4}{8} = \frac{1}{2}$$

Mid-point is $$x = \frac{2 + \frac{1}{2}}{2} = \frac{5}{4}$$

and $$y = \frac{4+2}{2} = 3$$

Hence, $$(x, y) = \left(\frac{5}{4}, 3\right)$$

10. (d) Let $P(h,k)$

$\therefore$ Equation of polar, $yk = 2a(x+h)$ touches

$$x^2 = 4by$$

$\Rightarrow$ $$x^2 = 4b \cdot \frac{2a}{k}(x+h)$$

$\Rightarrow$ $Kx^2 - 8abx - 8abh = 0$

Here, $$D = 0$$

$$64a^2b^2 - 4(k)\ (-8abh) = 0$$

$\Rightarrow$ $$32ab[2ab + hk] = 0$$

$\therefore$ $$hk = -2ab$$

$\Rightarrow$ $$xy = -2ab$$

Hence, locus is rectangular hyperbola.

11. (c) $x + y = 6$

$\therefore$ $y = -x + 6$ is normal to $y^2 = 8x$.

We know that the equation of normal to $y^2 = 4ax$ is

$$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$

$\Rightarrow$ Slope, $$-\frac{y_1}{2a} = -1$$

$\Rightarrow$ $$\frac{-y_1}{4} = -1 \quad [\because a = 2]$$

$\Rightarrow$ $$y_1 = 4$$

On putting $y_1 = 4$ in $y^2 = 8x$, we get $x = 2$

Hence, point is (2, 4).

12. (a) Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then, $$2a = 6b$$

$\Rightarrow$ $$\frac{b}{a} = \frac{1}{3}$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2} = \sqrt{1 - \frac{1}{9}}$$

$$= \sqrt{\frac{8}{9}}$$

$\Rightarrow$ $$e = \frac{2\sqrt{2}}{3}$$

13. (b) CP and CD are semi-conjugate diameters of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Obviously, $CP^2 + CD^2 = a^2 + b^2$

14. (c) On putting $y = mx + c$ in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$(a^2m^2 + b^2)x^2 + 2a^2cmx + a^2(c^2 - b^2) = 0 \quad \ldots(i)$$

$\therefore$ $$x_1 + x_2 = -\frac{2a^2cm}{a^2m^2 + b^2}$$

and $$x_1 \cdot x_2 = \frac{a^2(c^2 - b^2)}{a^2m^2 + b^2}$$

$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$$

$$= \frac{4a^4c^2m^2}{(a^2m^2 + b^2)^2} - \frac{4a^2(c^2 - b^2)}{a^2m^2 + b^2}$$

$$= \frac{4a^2b^2(a^2m^2 + b^2 - c^2)}{(a^2m^2 + b^2)^2}$$

Length of chord $$= \sqrt{\frac{(1+m^2)4a^2b^2(a^2m^2 + b^2 - c^2)}{(a^2m^2 + b^2)}}$$

So, $a^2m^2 \geq c^2 - b^2$ for real points.

15. (a) Equations of the curve are

$$x = a(\cosh\theta + \sinh\theta) \quad \ldots(i)$$

and $$y = b(\cosh\theta - \sinh\theta) \quad \ldots(ii)$$

On multiplying Eqs. (i) and (ii), we get

$$\frac{xy}{ab} = \cosh^2\theta - \sinh^2\theta$$

$\Rightarrow$ $$\frac{xy}{ab} = 1 \quad [\because \cosh^2\theta - \sinh^2\theta = 1]$$

$\therefore$ $$xy = c^2$$

Which is rectangular hyperbola.

16. (c) Ellipse is $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

and hyperbola is $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$\Rightarrow$ $$\frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

$\because$ Foci coincide

$\therefore \quad c = \sqrt{a^2 + b^2} = \sqrt{\left(\frac{12}{5}\right)^2 + \left(\frac{9}{5}\right)^2}$ [hyperbola]

$= \sqrt{\frac{225}{25}} = \sqrt{9} = 3$

Also, $c = \sqrt{a^2 - b^2} = \sqrt{16 - b^2}$ [ellipse]

$\Rightarrow \quad (3)^2 = 16 - b^2$

$\Rightarrow \quad b^2 = 7$

17. (a) We know that the equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = \pm x \pm \sqrt{a^2 - b^2}$.

As tangent is parallel to $y = x$.

$\therefore$ Slope is 1.

$\Rightarrow$ Equation is $y = x + \sqrt{3-2}$

$y = x + 1$

$\therefore \quad x - y + 1 = 0$

18. (b) Focus is (4, 0) and $e = \frac{4}{5}$.

We know that, $c = \sqrt{a^2 - b^2} = 4$

Now, $\frac{4}{5} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$ $\left[\because e = \sqrt{1 - \frac{b^2}{a^2}}\right]$

$\Rightarrow \quad \frac{16}{25} = 1 - \left(\frac{b}{a}\right)^2$

$\Rightarrow \quad \frac{b^2}{a^2} = \frac{9}{25}$

$\Rightarrow \quad b^2 = \frac{9}{25}a^2 = \frac{9}{25} \times 25$ $[\because c = ae]$

$\Rightarrow \quad b^2 = 9$

$\because \quad a^2 = 25$ and $b^2 = 9$

$\therefore$ Equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

19. (d) $f(x) = \log_x |\log x|$

$= \frac{\log_e |\log x|}{\log_e x}$ $\left[\because \log_a b = \frac{\log_e b}{\log_e a}\right]$

$= \frac{\log(\log x)}{\log x}$

$\Rightarrow f(x) = \frac{\log(\log x)}{\log x}$

$$f'(x) = \frac{\log x \times 1/\log x \times \frac{1}{x} - \log(\log x) \times \frac{1}{x}}{(\log x)^2}$$

$$\Rightarrow \quad f'(x) = \left(\frac{\frac{1}{x} - \frac{\log(\log x)}{x}}{(\log x)^2}\right)$$

Now, $f'(e) = \frac{\frac{1}{e} - \frac{\log(\log e)}{e}}{(\log e)^2} = \frac{e^{-1} - 0}{(1)^2} = e^{-1}$

20. (b) $y = x + e^x$

On differentiating w.r.t. y, we get

$$1 = \frac{dx}{dy} + e^x \cdot \frac{dx}{dy}$$

$\Rightarrow \quad \frac{dx}{dy} = (1 + e^x)^{-1}$

$\Rightarrow \quad \frac{d^2x}{dy^2} = -(1 + e^x)^{-2} \cdot e^x \cdot \frac{dx}{dy}$

$\Rightarrow \quad \frac{d^2x}{dy^2} = -e^x(1 + e^x)^{-3}$

21. (c) $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow \quad x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow \quad x^2(1+y) = y^2(1+x)$

$\Rightarrow \quad x^2 + x^2 y = y^2 + xy^2$

$\Rightarrow \quad (x^2 - y^2) = xy(y - x)$ $[y \neq x]$

$\Rightarrow \quad (x + y) = -xy$

$\Rightarrow \quad y = -\frac{x}{1+x}$

$\therefore \quad \frac{dy}{dx} = -\left[\frac{(1+x) - x}{(1+x)^2}\right] = \frac{-1}{(1+x)^2}$

22. (d) Let $y = \log\sqrt{\frac{1+\sin x}{1-\sin x}}$

$\Rightarrow \quad y = \frac{1}{2}[\log(1+\sin x) - \log(1-\sin x)]$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1+\sin x} \cdot \cos x + \frac{1}{1-\sin x}\cos x\right]$

$= \frac{1}{2}\left[\frac{\cos x - \cos x \sin x + \cos x + \cos x \sin x}{1 - \sin^2 x}\right]$

$= \frac{1}{2}\left[\frac{2\cos x}{\cos^2 x}\right]$

$= \sec x$

23. (d) $x = a\left[\cos t + \log\tan\frac{t}{2}\right]$ and $y = a\sin t$

$\Rightarrow \quad \frac{dx}{dt} = a\left[-\sin t + \frac{1}{2\tan\frac{t}{2}}\sec^2\frac{t}{2}\right]$ and $\frac{dy}{dt} = a\cos t$

$$\frac{dy}{dx} = \frac{a\cos t}{a\left[-\sin t + \frac{1+\tan^2\frac{t}{2}}{2\tan\frac{t}{2}}\right]} = \frac{\cos t}{-\sin t + \frac{1}{\sin t}}$$

$= \frac{\sin t \cdot \cos t}{1 - \sin^2 t} = \frac{\sin t \cdot \cos t}{\cos^2 t}$

$= \tan t$

24. (c) Slope of tangent to $y = x^2$ at (1, 1),

$$\left(\frac{dy}{dx}\right)_{x=1} = (2x)_{x=1} = 2 = m_1 \text{ [say]}$$

$\therefore$ Slope of tangent to the curve

$$y = 7 - \frac{1}{6}x^3 \text{ at } (1, 1),$$

$$\left(\frac{dy}{dx}\right)_{x=1} = \left(-\frac{1}{2}x^2\right)_{x=1} = -\frac{1}{2} = m_2 \quad \text{[say]}$$

As $\quad m_1 \times m_2 = -1$

$\therefore$ Angle of intersection is $\frac{\pi}{2}$.

25. (d) Given curve is $y - e^{xy} + x = 0$.

Then, slope of the tangent to the curve $y = x + e^{xy}$ at the point (a, b) is given by

$$\frac{dy}{dx} = 1 + e^{xy}\left[y + x \cdot \frac{dy}{dx}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}(1 - xe^{xy}) = 1 + y \cdot e^{xy}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=a} = \frac{1 + b \cdot e^{ab}}{1 - a \cdot e^{ab}}$$

The tangent is vertical, iff $1 - ae^{ab} = 0$

$$\Rightarrow \quad e^{ab} = \frac{1}{a}$$

$$\Rightarrow \quad a = 1 \text{ and } b = 0$$

26. (c) Given, $y = x^3 - 12x$

$$\frac{dy}{dx} = 3x^2 - 12$$

For maxima and minima,

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 3x^2 - 12 = 0$$

$$\Rightarrow \quad x = 2, -2, \quad 2 \in [0, 3]$$

$\frac{d^2y}{dx^2} = 6x = 6 \times 2 = 12$, which is positive.

Hence, the minimum value of $y = (2)^3 - 12 \times 2$

$$= 8 - 24$$

$$= -16$$

27. (b) Given,

$$u = x^2y + y^2z + z^2x$$

$$\frac{\partial u}{\partial x} = 2xy + z^2 \quad \ldots\text{(i)}$$

$$\frac{\partial u}{\partial y} = 2yz + x^2 \quad \ldots\text{(ii)}$$

$$\frac{\partial u}{\partial z} = 2zx + y^2 \quad \ldots\text{(iii)}$$

On adding Eqs. (i), (ii) and (iii), we get

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

$$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$= (x + y + z)^2$$

28. (a) $\int e^x\left[\frac{1}{x} - \frac{1}{x^2}\right]dx$

$$= e^x \times \frac{1}{x} + C$$

$$\left[\because \int e^x [f(x) + f'(x)]dx = e^x f(x) + C\right]$$

29. (c) Let $I = \int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx$

On putting $\quad \tan^{-1} x = u$

$$\Rightarrow \quad \frac{1}{1 + x^2} dx = du$$

When $x = 0$, then $u = 0$

When $x = 1$, then $u = \pi / 4$

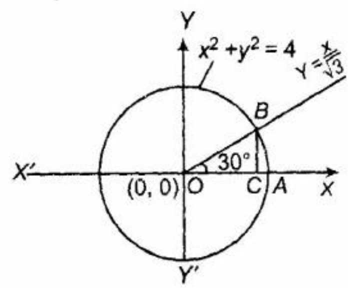
$$I = \int_0^{\pi/4} u \cdot du$$

$$= \left[\frac{u^2}{2}\right]_0^{\pi/4}$$

$$= \frac{\pi^2}{2 \times 16}$$

$$= \frac{\pi^2}{32}$$

30. (c)



On putting $y = \frac{x}{\sqrt{3}}$ in $x^2 + y^2 = 4$, we get

$$x^2 + \frac{x^2}{3} = 4$$

$$\Rightarrow \quad 4x^2 = 12$$

$$\Rightarrow \quad x = \pm\sqrt{3}$$

Required area $= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^2$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 2\left(\sin^{-1} 1 - \sin^{-1}\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

31. (d) $\int_0^1 \frac{dx}{1 + x}$

Given, $\quad n = 2$

$$\therefore \quad h = \frac{1}{2}$$

$x_0 = 0, \quad y_0 = 1$

$x_1 = \frac{1}{2}$, $\quad y_1 = \frac{2}{3}$

$x_2 = 1$, $\quad y_2 = \frac{1}{2}$

Now, $$\int_0^1 \frac{dx}{1+x} = h\left[\frac{1}{2}(y_0 + y_2) + y_1\right]$$
$$= \frac{1}{2}\left[\frac{1}{2}\left(1+\frac{1}{2}\right) + \frac{2}{3}\right]$$
$$= \frac{1}{2}\left[\frac{3}{4} + \frac{2}{3}\right]$$
$$= \frac{1}{2}\times\left(\frac{9+8}{12}\right)$$
$$= \frac{17}{24}$$

32. (c) Obviously, curve of degree is 2.

33. (b) Given, $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$

$$\Rightarrow \quad u_x = \frac{1}{\sqrt{1-\frac{(\sqrt{x}-\sqrt{y})^2}{(\sqrt{x}+\sqrt{y})^2}}}\cdot\frac{d}{dx}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$$
$$= \frac{(\sqrt{x}+\sqrt{y})}{\sqrt{2\sqrt{x}\cdot 2\sqrt{y}}}\times\frac{\frac{1}{2\sqrt{x}}\cdot 2\sqrt{y}}{(\sqrt{x}+\sqrt{y})^2}$$
$$u_x = \frac{\sqrt{y}}{2\sqrt{x}\sqrt{\sqrt{xy}}(\sqrt{x}+\sqrt{y})} \quad \ldots(\text{i})$$

Similarly, $$u_y = \frac{-\sqrt{x}}{2\sqrt{y}\sqrt{xy}(\sqrt{x}+\sqrt{y})} \quad \ldots(\text{ii})$$

On dividing Eq. (i) by Eq. (ii), we get
$$\frac{u_x}{u_y} = -\frac{y}{x}$$
$$\therefore \quad u_x = -\frac{y}{x}\cdot u_y$$

34. (c) Given,
$$\frac{dy}{dx} = \frac{3e^{2x}+3e^{4x}}{e^x+e^{-x}}$$
$$\Rightarrow \quad dy = 3\int\frac{(e^{2x}+e^{4x})\cdot e^x}{e^{2x}+1}dx$$
$$= 3\int\frac{(e^{2x}+1)\cdot e^{3x}}{e^{2x}+1}dx = 3\int e^{3x}dx$$
$$\therefore \quad y = e^{3x} + C$$

35. (b) Given, $\frac{dy}{dx} = \cos(x+y)$

On putting $x + y = z$
$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{dz}{dx} - 1$$

Now, $$\frac{dz}{dx} - 1 = \cos z$$
$$\Rightarrow \quad \frac{dz}{dx} = 1 + \cos z$$
$$\Rightarrow \quad \frac{1}{1+\cos z}\cdot dz = dx$$
$$\Rightarrow \quad \frac{1}{2}\int\sec^2\frac{z}{2}\cdot dz = \int dx$$
$$\Rightarrow \quad \frac{1}{2}\cdot\frac{\tan\frac{z}{2}}{\frac{1}{2}} = x + C$$
$$\Rightarrow \quad \tan\left(\frac{z}{2}\right) = x + C$$
$$\therefore \quad \tan\frac{(x+y)}{2} = x + C$$

36. (d) Given equation of line is $ax + by = 1$.

On differentiating w.r.t. x, we get
$$a + b\frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = -\frac{a}{b}$$

Again, on differentiating, $\frac{d^2y}{dx^2} = 0$
$$\therefore \quad \frac{d^2y}{dx^2} = 0$$

37. (c) Given differential equation is
$$(x+y)dx + xdy = 0$$
$$\Rightarrow \quad x + y = -x\frac{dy}{dx}$$
$$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{x}\cdot y = -1$$
$$\text{IF} = e^{\int\frac{1}{x}dx} = e^{\log x} = x$$

Then, the solution is
$$y\cdot x = \int -x\cdot dx + C$$
$$\Rightarrow \quad xy = \frac{-x^2}{2} + C$$
$$\therefore \quad x^2 + 2xy = C$$

38. (a) Given differential equation is
$$y\cdot\frac{dy}{dx} = x - 1$$
$$\Rightarrow \quad y\,dy = (x-1)dx$$
$$\Rightarrow \quad \int y\,dy = \int(x-1)dx$$
$$\Rightarrow \quad \frac{y^2}{2} = \frac{(x-1)^2}{2} + C$$

When $x = 1$, $y = 1$
$$\Rightarrow \quad 1 = C$$

Therefore, the solution is
$$y^2 = x^2 - 2x + 2$$

39. (c) $\frac{dy}{dx} + \frac{2}{x} y = 3x^2 y^{4/3}$

$$y^{-4/3} \frac{dy}{dx} + \frac{2}{x} y^{1-4/3} = 3x^2$$

$$\Rightarrow \quad y^{-4/3} \frac{dy}{dx} + \frac{2}{x} y^{-1/3} = 3x^2$$

$$y^{-1/3} = z$$

$$\Rightarrow \quad -\frac{1}{3} y^{-4/3} \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \quad -\frac{3dz}{dx} + \frac{2}{x} z = 3x^2$$

$$\Rightarrow \quad \frac{dz}{dx} - \frac{2}{3x} z = -x^2$$

$$IF = e^{\int \frac{-2}{3x} dx}$$

$$= e^{-\frac{2}{3}\log x}$$

$$= e^{\log(x)^{-2/3}}$$

$$= (x)^{-2/3}$$

40. (b) Given differential equation is

$$(x + y + 1)\frac{dy}{dx} = e^{x-y}$$

$$\Rightarrow \quad (x + y)dy + dy = (e^{x-y})dx$$

$$\Rightarrow \quad e^y[(x + y)]dy + e^y dy = e^x dx$$

$$\Rightarrow \quad \int d[x + y]e^y = \int e^x dx$$

$$\Rightarrow \quad (x + y)e^y = e^x + C$$

41. (c) $\sqrt{1+x^2} \cdot dy + \sqrt{1+y^2} \cdot dx = 0$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{1+y^2}}{1+x^2}$$

$$\Rightarrow \quad \int \frac{1}{\sqrt{1+y^2}} dy = -\int \frac{1}{\sqrt{1+x^2}} dx$$

$$\Rightarrow \quad \log[y + \sqrt{1+y^2}] = -\log[x + \sqrt{1+x^2}] + \log C$$

$$\Rightarrow \quad \log(y + \sqrt{1+y^2})(x + \sqrt{1+x^2}) = \log C$$

$$\therefore \quad (y + \sqrt{1+y^2})(x + \sqrt{1+x^2}) = C$$

42. (a) $a\left(x\frac{dy}{dx} + 2y\right) = xy\frac{dy}{dx}$

$$\Rightarrow \quad ax\frac{dy}{dx} + 2ay = xy\frac{dy}{dx}$$

$$\Rightarrow \quad (xy - ax)\frac{dy}{dx} = 2ay$$

$$\Rightarrow \quad x(y - a)\frac{dy}{dx} = 2ay$$

$$\Rightarrow \quad \int\left(\frac{y-a}{2ay}\right)dy = \int\frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2a}\int\left(1 - \frac{a}{y}\right)dy = \int\frac{1}{x}dx$$

$$\Rightarrow \quad \frac{1}{2a}(y - a\log y) = \log x + C$$

$$\Rightarrow \quad \frac{y}{2a} - \frac{\log y}{2} = \frac{\log x^2}{2} + \frac{\log C}{2}$$

$$\Rightarrow \quad \frac{y}{a} + \log C = \log x^2 + \log y$$

$$\therefore \quad \frac{y + C}{a} = \log y x^2 \Rightarrow yx^2 = e^{\frac{y+c}{a}}$$

43. (b) Given, $\left(x\sin\frac{y}{x}\right)dy = \left(y\sin\frac{y}{x} - x\right)dx$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{y \cdot \sin\frac{y}{x} - x}{x \cdot \sin\frac{y}{x}}\right]$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{vx \cdot \sin v - x}{x\sin v}$$

$$= \frac{v\sin v - 1}{\sin v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v \cdot \sin v - 1 - v\sin v}{\sin v} = -\frac{1}{\sin v}$$

$$\int \sin v \cdot dv = -\int \frac{1}{x} \cdot dx$$

$$\Rightarrow \quad -\cos v = -\log x - C$$

$$\therefore \quad \cos\frac{y}{x} = \log|x| + C$$

44. (b) $u_{x+3} - 5u_{x+2} = 2^x$

On putting $x = 0$, we get order of given differential equation is 3.

45. (a) $u_{x+2} - 7u_{x+1} + 10u_x = 12(4^x)$

The associated homogeneous equation is

$u_{x+2} - 7u_{x+1} + 10u_x = 0$

Two solutions of this equation are 5^x and 2^x.

These are linearly independent.

To find a solution of the original equation we can guess that it takes the form $u_x^* = a4^x$

In order for u_x^* to be a solution, we need

$$a4^{x+2} - 7 \times a \times 4^{x+1} + 10 \times a \times 4^x = 12(4^x)$$

$$\Rightarrow \quad 16a4^x - 28a4^x + 10a4^x = 12(4^x)$$

$$\Rightarrow \quad 16a - 28a + 10a = 12$$

$$\Rightarrow \quad -2a = 12$$

$$\Rightarrow \quad a = -6$$

Thus, $u_x^* = -6 \cdot 4^x$ is a solution.

We conclude that the general solution of the equation is

$u_x = C_1 2^x + C_2 5^x - 6 \cdot 4^x$

46. (*) None of the option is correct.

47. (d) Given, $e^{\frac{dy}{dx}} = x^x$

Taking log on both sides, we get

$$\frac{dy}{dx} = x\log x$$

$\Rightarrow \quad \int 1dy = \int x \cdot \log x \cdot dx$

$\Rightarrow \quad y = \frac{x^2}{2}\log x - \int \frac{x^2}{2} \cdot \frac{1}{x} \cdot dx$

$= \frac{x^2}{2} \cdot \log x - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\log x - \frac{x^2}{4} + C$

$\therefore \quad y = \frac{x^2}{2}\left(\log x - \frac{1}{2}\right) + C$

48. (c) Slope of the tangent at (x, y) to a curve is $-\left(\frac{y+3}{x+2}\right)$.

$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{y+3}{x+2}\right)$

$\Rightarrow \quad \int \frac{1}{y+3}dy = -\int \frac{1}{x+2}dx$

$\Rightarrow \quad \log(y+3) = -\log(x+2) + \log C$

$\Rightarrow \quad y + 3 = \frac{C}{(x+2)}$

$\Rightarrow \quad y = 0$, when $x = 0$

$\Rightarrow \quad 3 = \frac{C}{2}$

$\Rightarrow \quad C = 6$

$\therefore$ The equation of the curve is

$y + 3 = \frac{6}{(x+2)}$

$\Rightarrow \quad (x+2)(y+3) = 6$

$\Rightarrow \quad xy + 3x + 2y + 6 = 6$

$\therefore \quad 3x + 2y + xy = 0$

49. (c) We know that probability of an event A in a random experiment is always greater than or equal to 0 and less than or equal to 1.

i.e. $\quad 0 \le P(A) \le 1$

50. (c) It is quite obvious that probability of occurring exactly one of A and B is $P(A \cup B) - P(A \cap B)$.

51. (c) One letter (vowel) can be selected from A, O, I, I in 4 ways.

One letter out of 11 letters of 'PROBABILITY' can be selected in 11 ways.

$\therefore$ P (of the letter being vowel) $= \frac{4}{11}$

52. (a) There are 10 prime numbers between 1 and 30.

Hence, P(of the number being prime) $= \frac{10}{30} = \frac{1}{3}$

53. (b)

Let $AC = x$ km and $BC = 3x$ km

Time taken by $A = \frac{x}{10}$ and time taken by $B = \frac{3x}{20}$

$\therefore$ Average speed $= \frac{4x}{\frac{x}{10} + \frac{3x}{20}} = \frac{4x \times 20}{5x}$

$= 16$ km/h

54. (c) We know that, SD $= \sqrt{\Sigma|x - \bar{x}|}$

is unchanged with the change of origin.

$\therefore$ SD of $ax + b$ is $|a|\,\sigma$.

55. (d) We know that, mode is measure of central tendency r the measure of dispersion.

56. (b) Karl Pearson's coefficient of skewness

$= \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$

$\Rightarrow \quad 0.32 = \frac{39.6 - \text{Mode}}{6.5}$

$\Rightarrow \quad \text{Mode} = 39.6 - 2.08$

$= 37.52$

$\because$ Mode = 3 Median − 2 Mean

$\Rightarrow \quad 37.52 = 3\text{Median} - 2 \times 39.6$

$\Rightarrow \quad 3\,\text{Median} = 37.52 + 79.2$

$\Rightarrow \quad 3\text{Median} = 116.72$

$\Rightarrow \quad \text{Median} = \frac{116.72}{3}$

$= 38.906$

$= 38.91$

57. (c) We know that,

$\text{Variance} = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$= \frac{16.9}{10} - \left(\frac{12}{10}\right)^2$

$= 1.69 - 1.44 = 0.25$

58. (d) We know that,

Mean $= np$

Variance $= npq$

$np = 3$ and $npq = 2$

$\Rightarrow \quad q = \frac{2}{3}$ and $p = 1 - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \quad n = 9$

$\therefore$ Binomial distribution is $\left(\frac{2}{3} + \frac{1}{3}\right)^9$.

59. (b) We know that,

$$\rho(x, y) = \frac{\Sigma xy - \frac{\Sigma x \cdot \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} \cdot \sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

Given, $\quad \bar{x} = 5$ and $\bar{y} = 6$

$\because \quad n = 10$

$\therefore \quad \frac{\Sigma x}{n} = 5$ and $\frac{\Sigma y}{n} = 6$

Now,
$$\rho(x,y)=\frac{\frac{\Sigma xy}{n}-\frac{\Sigma x\cdot\Sigma y}{n^2}}{\sqrt{\text{Variance of }x}\cdot\sqrt{\text{Variance of }y}}$$
$$=\frac{\frac{350}{10}-\frac{50\times 60}{100}}{\sqrt{4}\cdot\sqrt{9}}=\frac{35-30}{2\times 3}=\frac{5}{6}$$

(c)

x	0	1	2	3
y	1	0	1	10

$$y=\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}\times(1)$$
$$+\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}\times(0)$$
$$+\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}\times(1)$$
$$+\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}\times(10)$$
$$=\frac{(x-1)(x^2-5x+6)}{-6}+\frac{x(x^2-4x+3)}{-2}$$
$$+\frac{10x(x^2-3x+2)}{6}$$
$$=-\left[\frac{x^3-6x^2+11x-6+3x^3-12x^2+9x-10x^3+30x^2-20x}{6}\right]$$
$$=-\left[\frac{-6x^3+12x^2-6}{6}\right]=\frac{6x^3-12x^2+6}{6}$$
$$\Rightarrow \quad y=x^3-2x^2+1$$
Now, $y(4)=4^3-2(4)^2+1$
$=64-32+1=65-32=33$

(d)

x_i	y_i	x_i^2	x_iy_i
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
10	16.9	30	47.1

$$a=\frac{(n\,\Sigma x_i\,y_i)-(\Sigma\,x_i\,\Sigma y_i)}{(n\,\Sigma x_i^2)-(\Sigma x_i)^2}$$
$$=\frac{5\times 47.1-10\times 16.9}{5\times 30-(10)^2}$$
$$=\frac{235.5-169}{150-100}$$
$$=\frac{66.5}{50}$$
$$=\frac{6.65}{5}$$
$$=1.33$$
and
$$b=\frac{(\Sigma x_i^2)(\Sigma y_i)-(\Sigma x_i)(\Sigma x_iy_i)}{(n\Sigma x_i^2)-(\Sigma x_i)^2}$$
$$=\frac{30\times 16.9-10\times 47.1}{5\times 30-(10)^2}$$
$$=\frac{507-471}{150-100}$$
$$=\frac{36}{50}=0.72$$
Hence, the best fit straight line is given as
$$y=1.33x+0.72$$

62. (b) $P\left(\frac{\overline{B}}{\overline{A}}\right)=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{A})}=\frac{P(A\cup B)'}{P(A)'}$
$$=\frac{1-P(A\cup B)}{1-P(A)}=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(A)}$$
$$=\frac{1-\left[\frac{3}{8}+\frac{1}{2}-\frac{1}{4}\right]}{1-\frac{3}{8}}=\frac{1-\frac{5}{8}}{5/8}=\frac{3}{5}$$

63. (c) By property of poisson distribution, mean and variance are equal.

64. (c) Set of instructions in a sequential manner telling the computer what to do is called a program.

65. (b) ALU performs simple maths for CPU.

66. (c) Window is not a hardware of a computer.

67. (c) CPU is called the brain of the computer.

68. (b) WWW stands for world wide web.

69. (c) Software used to carry out this task is application software.

70. (c) Arrow lock key is pressed to work as the directional arrow.

71. (a) $(2\,37)_8=(010\ \ 011\ \ 111)_2$

72. (c) 1 gigabyte = 1000,000,000 bytes

73. (c) LOTUS is not a computer language.

74. (b) A B I L I T Y

$\therefore$ Number of arrangements $=\frac{7!}{2!}$
$=2520$

75. (c) $73^2-41^2+29^2=(73)^2-(41^2-29^2)$
$=73^2-(41+29)(41-29)$
$=5329-70\times 12$
$=5329-840=4489$

76. (*d*) $\frac{58x}{100} - \frac{37x}{100} = 399 \Rightarrow 21x = 39900$

$\Rightarrow \quad x = 1900$

$\therefore \quad 72\%$ of $x = \frac{72 \times 1900}{100} = 1368$

77. (*a*)

$$6)4700(69$$
$$\underline{36}$$
$$129)1100$$
$$\underline{1161}\uparrow$$

61 should be added.

78. (*d*) Let the cost of pens and pencils be ₹x and ₹y respectively.

$\therefore \quad 10x + 12y = 138$

As number of pens and pencils are in ratio 3 : 2.

$\therefore$ Their total cost will be $138 \times \frac{3}{2} = 69 \times 3 =$ ₹ 207.

79. (*a*) $R \rightarrow 5, A \rightarrow 7, I \rightarrow 9, P \rightarrow 4$

$\therefore \quad PAIR \rightarrow 4795$

80. (*b*) It is clear by observing the word.

81. (*d*) $\frac{1}{4}\left(\frac{x}{5}\right) = 7$

$\Rightarrow \quad x = 140$

$\frac{3}{14}$ of $140 = \frac{140}{14} \times 3 = 30$

82. (*c*) Let number of hens and cows be x and y respectively, then

$x + y = 43 \quad \ldots(\text{i})$

$2x + 4y = 142 \quad \ldots(\text{ii})$

On solving Eqs. (i) and (ii), we get $x = 15$ and $y = 28$

Hence, number of hens is 15.

83. (*c*) Let the ages of Sita and Gita be $3x$ and $8x$.

$\frac{3x+7}{8x+7} = \frac{4}{9}$

$\Rightarrow \quad 27x + 63 = 32x + 28$

$\Rightarrow \quad 5x = 35$

$\Rightarrow \quad x = 7$

$\therefore$ Age of Gita $= 8 \times 7 = 56$ yr.

84. (*c*) Let A, B, C, D and E be $x, x+2, x+4, x+6$ and $(x+8)$ respectively, then

$\frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 45$

$\Rightarrow \quad 5x + 20 = 225$

$\Rightarrow \quad 5x = 205$

$\Rightarrow \quad x = 41$

$\because B = 43$ and $D = 47$, $\therefore B \times D = 43 \times 47 = 2021$

85. (*d*) We know that,

$$\text{CI} = P\left\{\left(1 + \frac{r}{100}\right)^n - 1\right\}$$

$$= 5500\left\{\left(1 + \frac{5}{100}\right)^2 - 1\right\}$$

$$= 5500\left\{\frac{441}{400} - 1\right\}$$

$$= \frac{5500 \times 41}{400} = 563.75$$

86. (*b*) $\frac{65}{100}x = 520$

$\Rightarrow \quad x = 800$

87. (*a*) We know that,

$$x^2 + y^2 = (x+y)^2 - 2xy$$
$$= (20)^2 - 2 \times 84$$
$$= 400 - 168 = 232$$

88. (*c*) Number of coins $= \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$

$$= \frac{\pi\left(\frac{4.5}{2}\right)^2 \times 10}{\pi\left(\frac{1.5}{2}\right)^2 \times 0.2}$$

$$= \frac{45 \times 45}{15 \times 15} \times \frac{100}{2}$$

$$= \frac{900}{2} = 450$$

89. (*c*) For $(a+b)^n$, $T_4 = {}^nC_3\, a^{n-3} \cdot b^3 \quad \ldots(\text{i})$

$T_{13} = {}^nC_{12}\, a^{n-12} \cdot b^{12} \quad \ldots(\text{ii})$

We know that, ${}^nC_r = {}^nC_{n-r}$

$\Rightarrow \quad {}^nC_{12} = {}^nC_{n-3}$

$\Rightarrow \quad n - 3 = 12$

$\therefore \quad n = 15$

90. (*b*) Let $S = 1 + 2x + 3x^2 + 4x^3 + \ldots$

$$\frac{xS}{(1-x)S} = \frac{x + 2x^2 + 3x^3 + \ldots}{1 + x + x^2 + x^3 + \ldots}$$

$$= 1 + \frac{x}{1-x} \qquad \left[S_\infty = \frac{a}{1-r} \text{ for infinite GP}\right]$$

$$= \frac{1}{1-x}$$

$\Rightarrow \quad S = \frac{1}{(1-x)^2}$

$\Rightarrow \quad [1 + 2x + 3x^2 + 4x^3 + \ldots]^{1/2} = \frac{1}{1-x} = (1-x)^{-1}$

$$= 1 + (-1) \cdot x + \frac{(-1)(-2)}{1.2} \cdot x^2 + \frac{(-1)(-2)(-3)}{1 \cdot 2 \cdot 3} x^3 + \frac{(-1)(-2)(-3)(-4)}{1 \cdot 2 \cdot 3 \cdot 4} x^4$$

$\therefore$ Coefficient of $x^4 = 1$

$$= \sum_{k=1}^{n} (-1)^k \cdot {}^nC_k = -{}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 - \dots (-1)^n\, {}^nC_n$$

We know that,

$$(1-x)^n = 1 - {}^nC_1 x + {}^nC_2 x^2 - {}^nC_3 x^3 + {}^nC_4 x^4 \dots (-1)^n\, {}^nC_n$$

On putting $x = 1$,

$$0 = 1 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 + \dots$$

$$\Rightarrow \quad -{}^nC_1 + {}^nC_2 - {}^nC_3 + \dots (-1)^n\, {}^nC_n = -1$$

(c) $\log(1 + x + x^2)$

$$= x + x^2 - \frac{(x+x^2)^2}{2} + \frac{(x+x^2)^3}{3} - \frac{(x+x^2)^4}{4} + \dots$$

$$= x + x^2 - \frac{1}{2}[x^2 + x^4 + 2x^3] + \frac{1}{3}[x^3 + x^6 + 3x^3(x + x^2)] \dots$$

$\therefore$ Coefficient of $x^3 = -1 + \dfrac{1}{3}$

$$= \frac{-2}{3}$$

Hence, coefficient of x^n is $\dfrac{-2}{n}$.

(c) $\dfrac{2}{1!} + \dfrac{6}{2!} + \dfrac{12}{3!} + \dfrac{20}{4!} + \dots$

$$= \frac{1\cdot 2}{1!} + \frac{3\cdot 2}{2!} + \frac{3\cdot 4}{3!} + \frac{4\cdot 5}{4!} + \dots$$

$$= \sum_{n=1}^{\infty} \frac{n(n+1)}{n!}$$

$$= \sum_{n=1}^{\infty} \frac{n^2}{n!} + \sum_{n=1}^{\infty} \frac{n}{n!}$$

$$= 2e + e$$

$$= 3e$$

(a) $x = \dfrac{1}{1\cdot 2} - \dfrac{1}{2\cdot 3} + \dfrac{1}{3\cdot 4} - \dfrac{1}{4\cdot 5} + \dots$

$$\Rightarrow x = \left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{4} - \frac{1}{5}\right) + \dots$$

$$\Rightarrow x = 1 - 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots\right)$$

$$= 1 + 2\left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots\right)$$

$$\Rightarrow \quad x = 1 + 2[\log_e 2 - 1]$$

$$\Rightarrow \quad e^x = e^{1 + 2\log_e 2 - 2}$$

$$= e^{-1} \cdot e^{\log_e 2^2}$$

$$= \frac{4}{e}$$

95. (b) In upper triangular matrix, the elements above main diagonal are not all zero.

96. (d) We know that, $(AB)^{-1} = B^{-1} \cdot A^{-1}$

97. (c) Given, $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Therefore, $\quad A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$

98. (d) Given, $\Delta = \begin{vmatrix} 1 & 3 & x \\ 1 & 1 & x^2 \\ 3 & 7 & 3 \end{vmatrix} = 0$

On applying $C_3 \to C_3 - C_1$,

$$\Delta = \begin{vmatrix} 1 & 3 & x-1 \\ 1 & 1 & x^2-1 \\ 3 & 7 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x-1)\begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & x+1 \\ 3 & 7 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x-1)\begin{vmatrix} 1 & 3 & 1 \\ 0 & -2 & x \\ 0 & -2 & -3 \end{vmatrix} = 0 \qquad \begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1 \end{bmatrix}$$

$$\Rightarrow \quad (x-1)(2x+6) = 0$$

$$x = 1 \text{ and } x = -3$$